On Minimal Supergravities in d>4

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Based on work with: HCT & Cumrun Vafa
Hamada & Montero & HCT & Cumrun Vafa

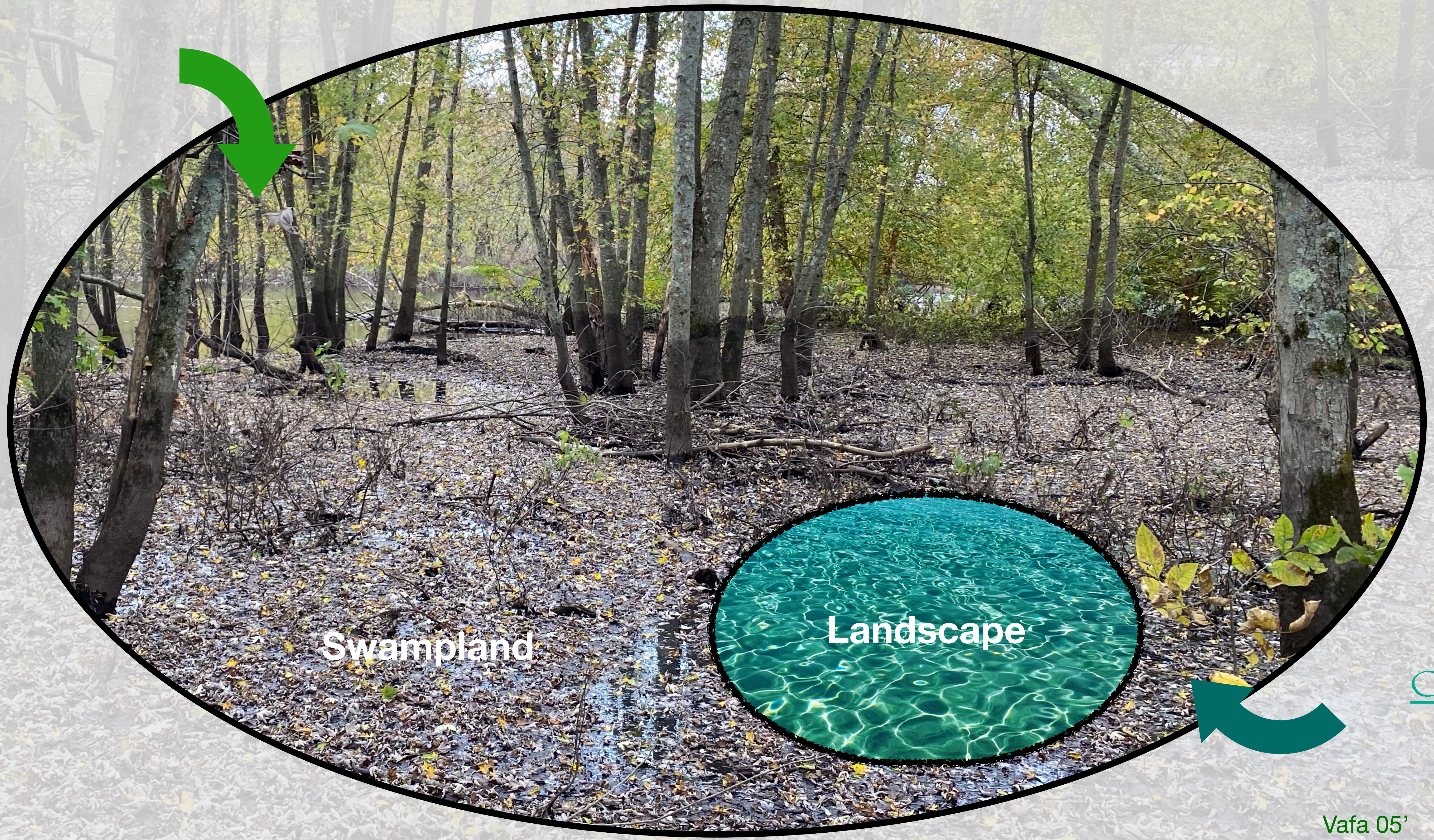
and previous work with: HCT & Cumrun Vafa (2106.10839)

Sheldon Katz, Hee-Cheol Kim , HCT, Cumrun Vafa (2004.14401)

Swampland Program

What conditions can we use to distinguish between
consistent QFT's which cannot couple to gravity and those that can arise in the low energy limit of a quantum gravity?

consistent QFT
Not Consistent with QG



consistent QFT
Consistent with QG

Vafa 05'


consistent QFT
Not Consistent with QG
Landscape
Swampland
String Landscape
consistent QFT
Consistent with QG
Do all the theories in the Landscape also belong in the String Landscape (SLP: YES)?

Is there a way to use <u>fundamental ideas</u> that are believed to be true for Quantum gravitational theories to construct such condition?

<u>Unitarity</u>

Every Quantum Gravity should be Unitary

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Completeness of Spectrum

In a quantum gravity for any p-form gauge field, every charged (p-1)-brane state should appear in the spectrum

Polchincki 03'
Banks, Seiberg 10'
Harlow, Ooguri 18'

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Cobordism Conjecture:

All the cobordism classes in a consistent theory of quantum gravity must vanish

McNamara,Vafa 19'

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McNamara,Vafa 19'

Distance Conjecture:

In a quantum gravity at infinite distance in the scalar moduli space an infinite tower of states becomes exponentially light

Ooguri, Vafa 06'

Many more Swampland conditions exist ....

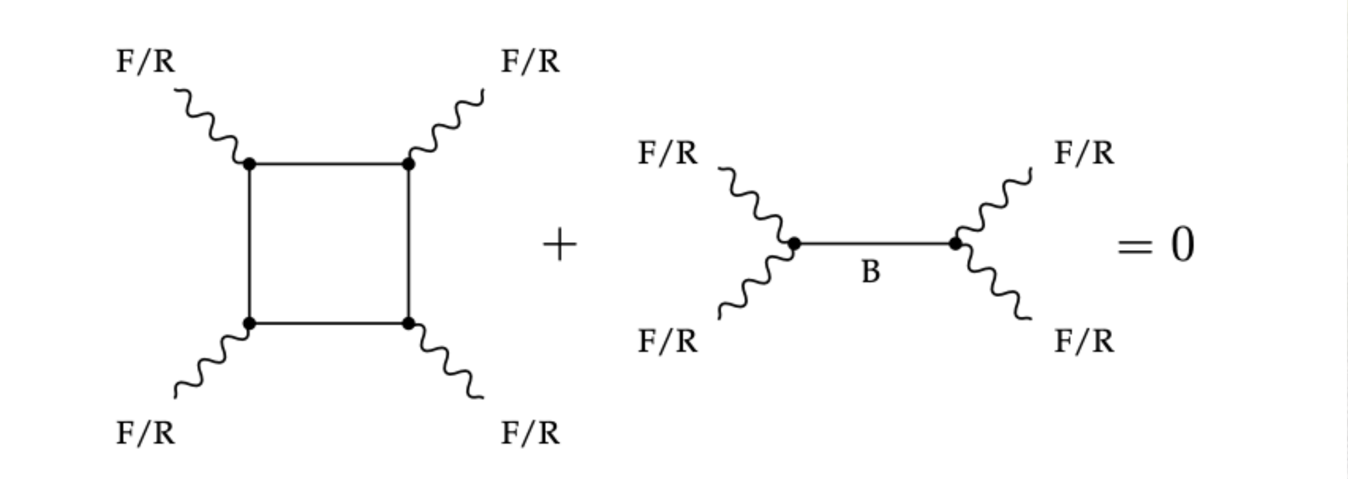
D	8 Supercharges	16 Supercharges	32 Supercharges
11	×	×	M-theory
10	×	Anomaly cancellation $G = E_8 \times E_8, SO(32), E_8 \times U(1)^{248}, U(1)^{496}$ [Kim, Shiu, Vafa 19'] [Adams, Dewolfe, Taylor 10']	IIA & IIB
9	×	$r \equiv 1 \mod 2$ [Alvarez-Gaume and E. Witten '84] $\cancel{Sp(n)}$ [Bedroya, Hamada, Montero, Vafa 21'] $r = 1$, M-theory on KB or IIB on DP $r = 9:$ CHL string $r = 17:$ Heterotic on S^1	$\downarrow$
8	×	$\downarrow S^1$ $r \equiv 0 \mod 2$ String Theory allows only for $8d: SU(N), SO(2N), Sp(N), e_6, e_7, e_8$ but not $\underbrace{SO(2N+1)}, \underbrace{f_4}, \underbrace{g_2}$ [Garc´ıa-Etxebarria, Hayashi ,Ohmori, Tachikawa, Yonekura 17'] [Hamada, Vafa 21']	$(S^1)^d$
7	×	$rank(G) = 1 \mod 2$ Montero, Vafa 20' $r = 3,5,7,11,19$ Can be realized.	
6	[Morrison, ,Kumar, Taylor,…..09'/10'/..] Various Conditions [Kim, Shiu, Vafa 19'] [Lee,Weigand 19'] [HCT, Vafa 21']	6d (2,0), which is unique and realized through IIB on K3 6d (1,1) $rank(G) \leq 20$ [Kim, HCT, Vafa 19']	
5	Various Conditions [Katz, Kim, HCT, Vafa 21']	$rank(G) \leq 21$ [Kim, HCT, Vafa 19']	$\downarrow$

6d $\mathcal{N} = 1$ Supergravity theories

Super-multiplets:

Supergravity-multiplet	$(g_{\mu\nu}, B_{\mu\nu}, \psi^-_\mu)$
Tensor-multiplet(T)	$(B_{\mu\nu}, \phi, \chi^+)$
Vector-multiplet(V)	(A_μ, λ^-)
Hyper-multiplet(H)	$(4h, \psi^+)$

Chiral fields contribute to gauge/gravitational anomalies
Cancelled by the **Green-Schwarz-Sagnotti Mechanism**



Anomaly polynomial factorizes as:

$$I_8 = \frac{1}{2}\Omega_{\alpha\beta}X_4^\alpha X_4^\beta$$

$$X^\alpha = \frac{1}{2}a^\alpha trR^2 + \sum_i b_i^\alpha(\frac{2}{\lambda_i}trF_i^2)$$

$\Omega_{\alpha\beta}$ symmetric of signature (1,T)

$a^\alpha, b_i^\alpha \in \mathbb{R}^{1,T}$

Anomaly Cancellation:

$R^4: \quad H - V = 273 - 29T$

$F^4: \quad 0 = B^i_{Adj} - \sum n^i_R B^i_R$

$(R^2)^2: \ a \cdot a = 9 - T \in \mathbb{Z}$

$F^2R^2: \ a \cdot b_i = \frac{1}{6}\lambda_i(A^i_{Adj} - \sum_R n^i_R A^i_R) \in \mathbb{Z}$

$(F^2)^2: \ b_i \cdot b_i = \frac{1}{3}\lambda_i^2(\sum_R n^i_R C^i_R - C^i_{Adj}) \in \mathbb{Z}$

$F_i^2F_j^2: \ b_i \cdot b_j = \sum_{R,S} \lambda_i\lambda_j n^{ij}_{RS} A^i_R A^j_S \ \in \mathbb{Z}, \ i \neq j$

A_R, B_R, C_R group theory coefficients

$tr_RF^2 = A_R trF^2, \ tr_RF^4 = B_R trF^4 + C_R(trF^2)^2$

$n^i_R =$ hypers in number of in R

[Taylor, Kumar, Morison,….]

The number of theories that satisfy these anomaly conditions are infinite.
SLP would though suggest that only a finite number of those can truly be UV-completed in a quantum gravity.

• 6d $\mathcal{N}=1$ Supergravity theories

Completeness of Spectrum

- For 6d $\mathcal{N}=1$ supergravity there exist (anti-)shelf dual BPS strings charged under B_2

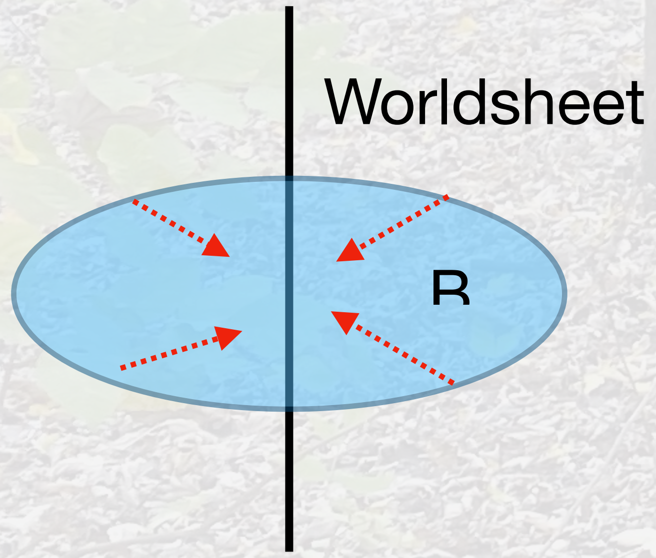
  → (0,4) SCFT at low energy

Unitarity of the string worldsheet

$$\boxed{\sum_i c_{G_i} = \frac{k_i \, dimG}{k_i + h_i^\vee} \leq c_L}$$

Anomaly cancellation for string of charge Q

$$\boxed{-I_4^{inflow} + I_4^{WS} = 0}$$



[Kim, Kim, Park 16']

[Shimizu, Tachikawa 16']

- **Central charges**: $\hat{c}_L = 3Q\cdot Q - 9Q\cdot a + 2$, $\hat{c}_R = 3Q\cdot Q - 3Q\cdot a \geq 0$
- **Levels of** G_i, $SU(2)_l$: $k_i = Q\cdot b_i$, $k_l = \frac{1}{2}(Q\cdot Q + Q\cdot a + 2) \geq 0$

$$\boxed{\text{Supergravity Strings } Q^2 \geq 0}$$

- The vertex operator of the massless modes with representation $\mathbf{R}$ of G with conformal weight $\Delta_{\mathrm{R}} = \frac{C_2(\mathbf{R})}{2(k+h^\vee)}$ where $C_2(\mathbf{R})$ is the second Casimir of the R must obey:

[HCT, Vafa'21]

$$\boxed{\Delta_{\mathbf{R}} \leq 1}$$

- The representation R of a primary with highest weight $\mathbf{\Lambda} = (\Lambda_i, \cdots, \Lambda_r)$ where r is the rank of the Lie algebra must satisfy :

$$\boxed{\sum_i^r \Lambda_i \leq k}$$

where k is the level of the current algebra of G on the worldsheet.

Is there a way to use fundamental ideas that are believed to be true for Quantum gravitational theories to construct such condition?

Emergent String Proposal

Infinite Distance Limit-
Emergence of
Tensionless String

+

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Assumption: The string lattice Λ is generated by Supergravity strings $Q_i^2 \geq 0,\ Q_i \cdot Q_j \geq 0$ [HCT, Vafa'21]

- $j \cdot b_i\ trF^2 \Longrightarrow j \cdot b_i > 0$
- $j \cdot j > 0$
- Tension: $Q \cdot j \geq 0$
- $-j \cdot a\ trR^2 \Longrightarrow j \cdot a > 0$

The moduli space of the theory is $SO(1,T)/SO(T)$ parametrized by $j \in \mathbb{R}^{1,T}$

Consider a theory with Gauge group G associated with vector b

Claim: in the limit $\frac{1}{g_{YM}^2} \sim j \cdot b \to +\infty$ while keeping $M_{pl}^2 \sim j^2$ finite

[Lee,Lerche,Weigand'18]

one can express $j = t\ Q_0 + \sum_i s_i\ Q_i$ in a basis of supergravity strings $\{Q_i\}$ with $Q_0^2 = 0$

where there must exist an asymptotically tensionless string Q_0 with

$$Q_0 \cdot b > 0$$

Unitarity on the String

$$-a \cdot Q_0 = 2$$

And

$$T_0 = j \cdot Q_0 \sim \frac{1}{t} \to 0 \quad \& \ T_b = j \cdot b \sim t \to +\infty \text{ as } t \to +\infty$$

Heterotic string of [Lee,Lerche,Weigand'18]

- Asymptotically tensionless string with $Q_0^2 = 0$ & $Q_0 \cdot b > 0$ [Lee,Lerche,Weigand'18]
- "Zariski Decomposition": unique way to write v=P+N [Bauer, Caibăr, Kennedy '09]

[Kumar, Morrison, Taylor 20']

Example 1: $SU(N)$ with $(N-8)$ F + 1 S with $T \leq 10$ and $N \leq 30$ (with $N = 30$ for $T = 1$) ← No F-theory realisation for any N

This theory has $-a \cdot b = -1$, $b \cdot b = -1$ therefore $-a = m\, b + N$ with $m > 0$ which implies that $(-a-b) \cdot Q_0 \geq 0 \Longrightarrow 2 \geq k = Q_0 \cdot b > 0$

But symmetric matter has highest weight $(2,0,\cdots,0)$.

$$\boxed{\sum_i^{N-1} \Lambda_i \leq 1}$$

Independent of choice of vectors $a,\ b,\ Q_0$

with $T = 1$ unique solution

[HCT, Vafa 21']

[Angelantonj, Bonnefoy, Condeescu, Dudas 20']

Example 2: $SU(24) \times SO(8)$ with 3 (A,1), $T = 1$ ← No F-theory realisation for any N

With $a^2 = 8,\ a \cdot b_1 = 3,\ a \cdot b_2 = -1,\ b_1^2 = 1,\ b_2^2 = -1,\ b_1 \cdot b_2 = 0$

Linear algebra implies : $Q \cdot b_1 \neq 0$

Unitarity: $\dfrac{k_1\,(24^2 - 1)}{k_1 + 24} > 20$

Generally, any gauge group with $b^2 > 0$ is restricted by unitarity since $Q \cdot b > 0$

E.g. for general $SU(N)$

N	$k \leq$
5	25
6	8
7	5
8	3
9	3
10	2
11	2

N	$k \leq$
12	1
13	1
14	1
15	1
16	1
17	1
18	1
19	1
20	1
21	1

Matter: $\sum_i^{N-1} \Lambda_i \leq 1$

Example 3: $SU(N) \times SU(N), T = 9$

With $a^2 = 0,\ b_1^2 = -2,\ b_2^2 = -2,\ b_1 \cdot b_2 = 2,\ a \cdot b_{1,2} = 0$

Linear algebra shows that $-a = m(b_1 + b_2),\ m \in \mathbb{Q}^+$

Gauge Group	Matter content
$SU(N) \times SU(N)$	$2(\square, \bar{\square})$
$SO(2N+8) \times Sp(N)$	$(\square, \square)$
$SU(N) \times SO(N+8)$	$(\square, \square) + (\begin{smallmatrix}\square\\ \square\end{smallmatrix}, 1)$
$SU(N) \times SU(N+8)$	$(\square, \square) + (\begin{smallmatrix}\square\\ \square\end{smallmatrix}, 1) + (1, \square\!\square)$
$Sp(N) \times SU(2N+8)$	$(\square, \square) + (1, \square\!\square)$

[Dabholkar and J. Park'96]

But $-a \cdot Q = 2$, hence $2 = m\ (k_1 + k_2)$ and $m = 1$ since a characteristic vector

Unitarity: $\dfrac{k_1\left(N^2-1\right)}{k_1+N} + \dfrac{k_2\left(N^2-1\right)}{k_2+N} \leq 20$

$\boxed{(k_1, k_2) = (2,0),\ (1,1):\ \ N \leq 11}$ $\longleftarrow$

String realisation: $N \leq 8$

For a specific choice: $N \leq 9$

[Shiu, Kim, Vafa'19]

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$Sp(N) \times SU(2N+8)$	$(\square, \square) + (1, \square\!\square)$

Similar analysis shows that they are all finite for independent of the choice of vectors!

- There are in fact more theories allowed by anomalies with k number of simple factors and excluded with similar tools as above:

$$Sp((N-8)/2) \underbrace{\times}_{F\otimes F} SU(N) \underbrace{\times}_{F\otimes F} SU(N+8) \times \cdots \underbrace{\times}_{F\otimes F} SO(N+8(k-2))$$

$$\underset{A}{SU(N-8)} \underbrace{\times}_{F\otimes F} SU(N) \underbrace{\times}_{F\otimes F} SU(N+8) \times \cdots \underbrace{\times}_{F\otimes F} \underset{S}{SU(N+8(k-2))}$$

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$$SU(N) \underbrace{\times}_{F\otimes F} SU(N) \times \cdots \underbrace{\times}_{F\otimes F} SU(N)$$

$F \otimes F$

Minimal Matter

D	8 Supercharges	16 Supercharges	32 Supercharges
11	x	x	M-theory
10	x	Anomaly cancellation $G = E_8 \times E_8, SO(32), E_8 \times U(1)^{248}, U(1)^{496}$	IIA & IIB
9	x	$r = 1$, M-theory on KB or IIB on DP $\downarrow$	
8	x	S^1	$(S^1)^d$
7	x	$rank(G) = 1 \mod 2$ Montero, Vafa 20'	
6	$H - V = 273 - 29T$	6d (2,0), unique, IIB on K3 $T = 21$ 6d (1,1) Asymmetric Orbifold: $\Gamma^{4,4}(A_4)$ with $\mathbb{Z}_5$ twist	
5	??????	5d $\mathcal{N} = 2$ Asymmetric Orbifold : $\Gamma^{5,5}(D5)$ with $\mathbb{Z}_{12}$ twist	

Gravity multiplet:	$(g_{\mu\nu}, A_\mu, 2\psi_\mu)$
Vector multiplet:	$(A_\mu, 2\chi, \phi)$
Hypermultiplet:	$(2\psi_-, 4\phi)$

← **No Dilaton!**

cannot be realized as low-energy theories of any perturbative string compactifications

Bosonic Action

$$S = \int \star R + \frac{1}{4} F \wedge \star F + \frac{c}{6} A \wedge F \wedge F + \frac{\kappa}{96} A \wedge R \wedge R$$

$$c \in \mathbb{N} \; \& \; \kappa \in 2\mathbb{Z}$$

How can one find such a theory?

Similar to M-theory so could one find a 4d $\mathcal{N} = 2$ supergravity with one vector multiplet(coming from the 5d reduction) whose strong coupling limit is the pure 5d theory?

[Mizoguchi '01]

Attempt: Non-geometric compactification using some asymmetric orbifold

Weyl group elements	Eigenvalues	Order
$E_6(a_1)$	$(\epsilon^1, \epsilon^2, \epsilon^4, \epsilon^5, \epsilon^7, \epsilon^8)$	9
$A_4 \oplus A_2$	$(\epsilon^3, \epsilon^6, \epsilon^9, \epsilon^{12}, \epsilon^5, \epsilon^{10})$	15
$D_4(a_1) \oplus A_2$	$(\epsilon^3, \epsilon^3, \epsilon^9, \epsilon^9, \epsilon^4, \epsilon^8)$	12
$A_2 \oplus A_1 \oplus A_1 \oplus A_1 \oplus A_1$	$(\epsilon^2, \epsilon^4, \epsilon^3, \epsilon^3, \epsilon^3)$	6

DID NOT WORK!

All the examples attempted had extra states in the twisted sectors

In fact it should not work!

The reason is rather simple!

The exact statement is: The 5d pure supergravity can not be a strong coupling limit of 4d perturbative string.

Consider the 5d action with 5d Planck mass M_5 being the only scale in the theory then the action is given by

$$S = M_5^3 \int R_5$$

And the tension of the string magnetically charged under the graviphoton is

$$T_5 = \frac{1}{2} M_5^2$$

Meanwhile, in 4d the supergravity action would be given by:

$$S = M_4^2 \int R_4$$

Comparing now this with the action of the 5d theory on a circle S^1 of radius L leads to

$$S = M_5^3 L \int R_4 \qquad\qquad M_5^3 L = M_4^2$$

By the dimensional reduction, the monopole string must become 4d BPS string.

The tensions of 4d BPS strings are

$$T_4 = \begin{cases} M_s^2 & \text{F-string} \\ \dfrac{M_s^2}{g_s} & \text{D-string} \end{cases} \qquad \begin{cases} L = \dfrac{M_s^2}{g_s^2 M_5^3} = \dfrac{1}{g_s^2 M_s} \\ L = \dfrac{1}{\sqrt{g_s} M_s} \end{cases}$$

Swampland or Landscape?

Anomalies

Swampland Conjectures

Self-Consistency

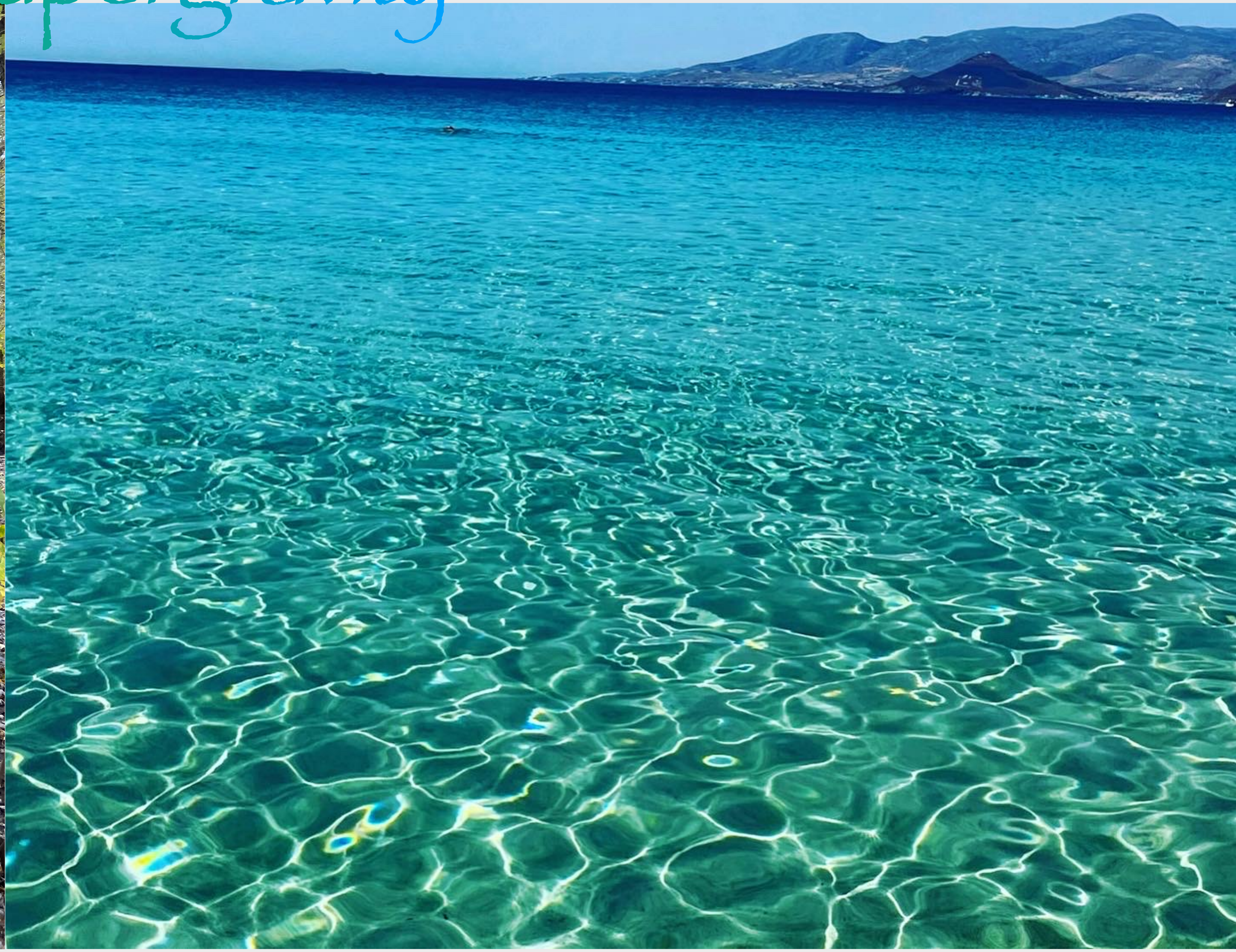
Possibilities ? ?

Existence of Holographic Dual ? ?

Connectedness to string theory ?

Proof by Vote
5d Minimal Supergravity

Boston,MA



Naxos, Greece

Thank you very much!